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LETTER TO THE EDITOR

The one-dimensional Ising model with a random transverse field at $T = 0$ from a real-space renormalisation group method

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Abstract. The one-dimensional Ising model in a random transverse field (taking the values Γ_1 and Γ_2 with probabilities p and $1-p$) is studied at $T=0$ from a real-space renormalisation group method. The phase diagram $f(\Gamma_1, \Gamma_2, p)$ is obtained and agrees with exact results.

The one-dimensional Ising model in a transverse field with Hamiltonian

$$H = - \sum_{\substack{i=1 \\ N\to\infty}}^{N} \Gamma_i s_i^z + J_i S_i^x S_{i+1}^x \tag{1}$$

where S_i^x and S_i^z are Pauli matrices

$$S_i^x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \text{and } S_i^z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

has been studied exactly for the non-random case (Pfeuty 1970) ($\Gamma_i = \Gamma$, $J_i = J$). The non-random model presents a transition at $T=0$. For Γ/J less than a critical value $(\Gamma/J)_c = 1$, the ground state $|0\rangle$ is degenerate and $M^x = \langle 0|S_i^x|0\rangle$ is $\neq 0$; when Γ/J goes to one, M^x goes to zero with a power law $M^x = [1-(\Gamma/J)^2]^{1/8}$ and for $\Gamma/J > 1$ the ground state is a singlet and $M^x = 0$. This $d=1$ quantum model at $T=0$ maps on the $d=2$ classical Ising model (Suzuki 1976) with horizontal exchange energies J_1 and vertical ones J_2 in the limit $J_1 \to 0$, $J_2 \to \infty$ and $(J_1/kT)/\exp(-J_2/kT) \to \Gamma/J$.

This non-random model has also been studied recently by a real space renormalisation method (Jullien *et al* 1978) well adapted to study quantum lattice systems at $T=0$. The results are in good agreement with exact results.

The random model (1) where Γ_i and J_i are different and satisfy certain probability distributions has never been studied carefully. Some exact results have been given recently by one of us (Pfeuty 1979): for any set of Γ_i and J_i the ground-state energy fails to be analytic if the condition

$$\pi_i \Gamma_i = \pi_i J_i \tag{2}$$

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is satisfied. This 'critical point' may not be unique. This random 1D quantum model at $T=0$ corresponds to a two-dimensional classical random Ising model at finite temperature: the Ising model with random infinitely long row defects studied by McCoy and Wu (1973), which constitutes the first random phase transition model to be studied exactly.

In this Letter we present preliminary results obtained by extending to the random model (equation 1) the renormalisation group method, developed for the non-random case (Jullien *et al* 1978). This constitutes the first attempt to study a random quantum system from a real-space renormalisation group method.

The simplest random case we case we consider here is a random chain (equation 1) with probability distributions for J_i and Γ_i

$$P(J_i)=\delta(J_i-J) \quad \text{and } P(\Gamma_i)=p\delta(\Gamma_i-\Gamma_1)+(1-p)\delta(\Gamma_i-\Gamma_2). \tag{3}$$

The Hamiltonian (1) defined on a 1D chain is separated into inter- and intra-block parts (the blocks are made of $n_s=2, 3, \ldots,$ sites). The transverse field Ising model is solved exactly for the block part with fields Γ_i satisfying (3) and the two lowest states $|+\rangle$ and $|-\rangle$ are retained. A new set of spin operators S'^{α} is associated with each block (the eigenstates of S'^{z} are $|+\rangle$ and $|-\rangle$) and the interblock parts are rewritten in the new spin representations. The new transverse field Γ'_i of the block (which is the difference $(E_i^+-E_i^-)/2$ where E_i^+ and E_i^- are the energies associated with the states $|+\rangle$ and $|-\rangle$) satisfy now an m delta function distribution ($m=3$ for $n_s=2$; $m=6$ for $n_s=3$). The probability distribution $P'(J'_i)$ for the new interaction parameter J'_i is now a set of delta functions (10 delta functions for $n_s=2$). The number of delta functions for the probability distribution increases very rapidly and it is much simpler and usually adequate to force the renormalised distribution $P'(J'_i)$ and $P'(\Gamma'_i)$ to take the same form as the initial distributions (3). This method has previously been used to study other disordered systems (Yeomans and Stinchcombe 1979) where it has been shown to give good results. The new parameters Γ'_1, Γ'_2, J' and p' are obtained by matching moments (the first moment for $P'(J'_i)$ and the first and second moments for $P'(\Gamma'_i)$). After one iteration one is left with a chain where the length has been divided by n_s with the same Hamiltonian (1) and with the same probability distributions (3) but with a new set of parameters $p'\Gamma'_1$, Γ'_2 J' which are functions of p, Γ_1, Γ_2, J. Our parameter space consists of three parameters Γ_1/J, Γ_2/J and p. The renormalisation group procedure gives a phase diagram. Figure 1 represents its projection on the Γ_1/J Γ_2/J plane for different values of p. The critical curves separating the ordered phase (doublet ground state) from the disordered phase (singlet ground state) can be described by $(\Gamma_1/J)^p$ $(\Gamma_2/J)^{1-p}\sim 1{\cdot}3$ and $(\Gamma_1/J)^p$ $(\Gamma_2/J)^{1-p}\sim 1{\cdot}15$ for $n_s=2$ and for $n_s=3$ respectively. This result is in good agreement with the exact result (Pfeuty 1979) $(\Gamma_1/J)^p$ $(\Gamma_2/J)^{1-p}=1$. The discrepancies are essentially due to the small size of the blocks and the results improve when n_s increases.

This first stage of approximation we have considered here is not yet sufficient when studying the critical behaviour. In fact, we find a new nontrivial fixed point different from the non-random fixed point ($\Gamma_1/J=1{\cdot}291$, $\Gamma_2/J=1{\cdot}088$, $p=0{\cdot}357$ for $n_s=3$)l. The non-random fixed point is unstable with a crossover exponent close to zero. The critical exponent ν for the new random fixed point is almost the same as for the non-random case. These results do not agree with what is expected both from the Harris argument (Harris 1974) $\phi=(2-d\nu)/\nu=(2-1)/1=1$ ($d=1$, $\nu=1$)) and from the exact results of McCoy and Wu (1973). This is due to the way we introduce the disorder in our renormalisation group method. After one iteration the probabilities

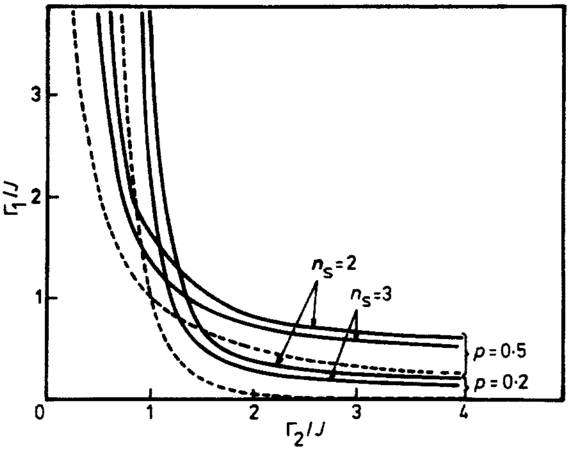

Figure 1. Critical curves $f(\Gamma_1/J, \Gamma_2/J)=0$ for p fixed $=0{\cdot}5$, $0{\cdot}2$ obtained from the renormalisation group method with $n_s=2$ and $n_s=3$. The broken curves correspond to the exact results.

$P'(\Gamma_i')$ and $P'(J_i')$ are *correlated*, and correlations cannot be neglected if one wants to consider the type of disorder of the Wu and McCoy model. By neglecting the correlations we have studied a model closely related to the bond-disordered two-dimensional Ising model. This partly explains why we found $\phi \sim 0$ and the same critical behaviour. The phase diagram is less affected by the drastic approximation for the probability distribution functions. We are now including these correlations by introducing two more parameters into the problem.

The same analysis is also being extended to $d=2$ for which the non-random case has already been studied (Penson *et al* 1979) and where no exact results exist at the moment. We are especially interested by the dilute case (Elliott and Saville 1974, Jullien *et al* 1979).

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